PREFACE TO THE 2017 PRINTING OF *GRAVITATION*

CHARLES W. MISNER AND KIP S. THORNE

As we look back on our sixty-year love affair with Einstein's general relativity, our primary emotion is joy: joy at having participated in an amazingly fruitful era of exploration and transformation. In geologists' terminology, we have lived a blessed bit of the Anthropocene epoch from a favored perch in the world, seeing wonders, while, fortunately, avoiding personally the wars and devastations that have afflicted so many others.

There is an immense contrast in human understanding of gravity in action from the 1950s, when John Wheeler recruited us into Einstein's arena, to the present time. In the 1950s, curved spacetime was a complex though beautiful way to interpret one observational datum from each of four phenomena: the bending of light by the Sun, the perihelion motion of Mercury, the gravitational redshift from the white dwarf 40 Eridani B, and the expansion of the universe. Today we have observational data by the megabyte. The icons for these data are (1) the WMAP-based plot of the variations of temperature of the cosmic microwave radiation as a function of angular scale—the marker for the advent of precision cosmology; and (2) the "chirp" plots of LIGO's first directly observed gravitational wave, marking the advent of gravitational wave astronomy. Along with these icons, there has been a wealth of other great insights and discoveries as the general relativity community expanded from a few dozen to a few thousand during the six decades since 1952, when John Wheeler began dreaming of this textbook.

THE CONTEXT IN WHICH WE WROTE *GRAVITATION*

General relativity had an exciting first two decades (1915–1939) and then became a two-decade backwater for physicists (1939–1958), as nuclear physics, elementary particle physics, and condensed matter physics came to the fore. In parallel, in mathematics, the field now called differential geometry was blossoming. For example, the concept of a manifold was clarified in the decades of the 1930s through the 1950s, and Milnor (1956) placed a capstone on this progress when he showed, by an example with the seven-dimensional sphere, that two manifolds that are equivalent at the level of continuous functions could be different in an essential way at the level of differentiable functions.

We were fortunate to enter relativity near the beginning of a remarkable renaissance (ca. 1958–1978), one enabled in part by the new mathematics and driven initially, in large measure, by our mentor John Wheeler, and then driven by a sequence of astronomical discoveries: the cosmic microwave background (CMB), and phenomena associated with black holes and neutron stars: quasars, pulsars, jets from galactic nuclei, compact X-ray sources, and gamma-ray bursts. It was late in this renaissance that we wrote *Gravitation*.

The relativity textbooks that preceded *Gravitation* were too old to incorporate the wonderful new observations and the new mathematical underpinnings. They treated Riemannian geometry as Einstein and then Pauli (1921) had, with almost no concept of the idea of a topological manifold that could carry properties (such as tangent vectors and 1-forms) even though it had been assigned no metric. They also, then, used no idea of points in the manifold (events in spacetime) as being conceptually superior to the various lists of coordinates used to identify them. And these texts tended to describe the physics almost entirely in terms of (old-fashioned) mathematics, with little attention paid to the heuristic but powerful tools by which modern physicists make rapid progress: physical arguments and pictures, geometric diagrams, and intuitive viewpoints. In *Gravitation*, our goal was to present relativity in physicists' physical, visual, and intuitive language, accompanied by the modern mathematics from which this language springs. The result was an advanced textbook with a far larger word-to-equation ratio than anything ever before seen in this field; a book filled with "purple prose," as John's wife, Janette, referred to it. But a book that also teaches relativity's mathematical underpinnings.

With our purple prose and pictures, we sought to transform how scientists think about relativity. And we think we succeeded, at least to some degree.

GRAVITATION'S GEOMETRIC VIEWPOINT

A major part of our approach is the geometric viewpoint on general relativity that we learned from John—a viewpoint that contrasts starkly with the field-theoretic viewpoint taken by Steven Weinberg in the relativity textbook (Weinberg 1972) that he wrote in parallel with our writing *Gravitation*.

For situations where spacetime is strongly curved and where we focus on regions comparable to or larger than its radius of curvature (e.g., black holes and a closed model universe), this geometric viewpoint is essential, or at least superior. For the causal structure of spacetime (horizons, singularities, Hawking's second law of black hole mechanics), it is also essential. For most other situations, while not essential, it is powerful. And whenever field-theoretic techniques are more useful than geometry (e.g., in the evolution of structure in the early universe), one can easily descend from the heights of geometry to the nitty gritty of field theory. (OK. Our prejudice is showing. Starkly.)

After decades steeped in the geometric viewpoint, one of us (Kip) has become so enamored of it that, with Stanford astrophysicist Roger Blandford, he has crafted a much broader textbook permeated with this viewpoint: a book titled *Modern Classical Physics* (Thorne and Blandford 2017; henceforth "MCP"), which covers all the areas of classical physics that PhD physicists should be exposed to but often are not, at least in North America. That book and this reprinting of *Gravitation* are being published simultaneously by the same publisher, Princeton University Press.

HOW USEFUL CAN *GRAVITATION* BE TODAY?

Gravitation was published in 1973, near enough to the end of the Relativistic Renaissance that most of that Renaissance's major theoretical insights and observational discoveries were in hand. While there have been some major additional insights and discoveries in the four decades since, they are few enough that *Gravitation* is seriously out of a date in only a moderate number of areas; primarily cosmology (Part VI), gravitational waves (Part VIII), experimental tests of general relativity (Part IX), and observations but not the theory of black holes and neutron stars (Parts V and VII).

This may account, in part, for *Gravitation*'s longevity: it continues to be used as supplemental reading in a large number of relativity courses around the world even today, 44 years after its publication. And in recent years, it has still been the primary textbook for a few courses.

CHAPTER-BY-CHAPTER STATUS OF *GRAVITATION*

As an aid to students, teachers, and other readers as they choose a path through relativity in the modern era, we offer here a chapter-by-chapter description of what in *Gravitation* is out of date and what is not; what is missing that we think so important that we would include it in a full year, advanced course in general relativity if we were teaching one; and where readers can go to learn about the missing developments.

1. **Parts I, II, III, and IV, the fundamentals of general relativity**, have not changed significantly over the past 44 years, so Chapters 1–22 on the fundamentals are almost fully up to date. The only exceptions are the following.
   A. *Chapter 8, Differential Geometry*, should be augmented by an introduction to symbolic manipulation software (e.g., *Maple*, *Mathematica*, and *Matlab*) for computing connection coefficients and curvature tensors and performing other tensorial calculations; and *Chapter 14, Calculation of Curvature*, could be augmented by a deeper treatment of symbolic manipulation.

B. To *Part IV, Einstein's Geometric Theory of Gravity*, we would add four new topics:
   a. *Numerical relativity*, which underpins gravitational wave observations and is teaching us about the nonlinear dynamics of curved spacetime; for example, Maggiore (2017), or for far greater detail, Baumgarte and Shapiro (2010) and Shibata (2016).
   b. *Gravitational lensing*, which is based on the linearized approximation to general relativity (Section 18.1) and has become a major tool for astronomy; for example, MCP or Straumann (2013), or for far greater detail, Schneider, Ehlers, and Falco (1992).
   c. *The Einstein field equation in higher dimensions*, particularly four space dimensions and one time dimension, which is motivated by string theory's requirement for higher dimensions and by the Randall-Sundrum (1999a,b) insight that one or more of these higher dimensions could be macroscopic. This topic often goes under the name "Braneworlds." For a brief treatment see, for example, Zee (2013); for much greater detail at the level of *Gravitation*, see Maartens and Koyama (2010).
   d. *Quantum field theory in curved spacetime* (which could be added at the end of Chapter 22). This topic underpins, most importantly, Hawking radiation from black holes; see below. For a brief introduction see, for example, Carroll (2004); for more thorough treatments, see Wald (1994) and Parker and Toms (2009).

2. **Part V, Relativistic Stars**, is similarly almost fully up to date, with the following two exceptions.
   A. *Chapter 24, Pulsars and Neutron Stars; Quasars and Supermassive Stars*, is completely out of date. Observations and observation-driven astrophysical theory have transformed our understanding profoundly. See, for example, Straumann (2013) or Maggiore (2017) or, for far greater detail, Shapiro and Teukolsky (1983), which is somewhat out of date but excellent and thorough.
   B. *Chapter 25* on geodesic orbits in the Schwarzschild spacetime should be augmented by exercises on computing orbits numerically to give the reader physical insight—which is best done by numerically integrating the Hamilton equations that follow from the super-Hamiltonian (Exercise 25.2); see Levin and Perez-Giz (2008).

3. **Part VI, The Universe**, is for the most part tremendously out of date.
   A. *Chapter 27, Idealized Cosmologies*, is an exception. The fundamental ideas and equations for idealized cosmologies have not changed, but the emphasis of this chapter is archaic. John, our mentor—whose intuition and prescience

were usually superb (Misner, Thorne, and Zurek 2009)—was firmly convinced that our universe would turn out to be closed and have vanishing cosmological constant; so in *Gravitation*, the closed Friedman cosmology is given great emphasis. Since *Gravitation* was published, a rich set of cosmological observations has revealed that our universe is very nearly flat spatially and has a positive cosmological constant (or something resembling it). So this chapter should be augmented by a more detailed treatment of the material in Section 27.11, and most importantly, by an in-depth treatment of the de Sitter solution of the Einstein equation with cosmological constant—as, for example, in Hawking and Ellis (1973). As a side issue (a Box), we would add the anti–de Sitter (AdS) solution (e.g., Hawking and Ellis 1973), because of its importance today in explorations of fundamental physics (e.g., the AdS/CFT correspondence).

B. *Chapter 28, Evolution of the Universe into Its Present State*, and *Chapter 29, Present State and Future Evolution of the Universe*, are completely out of date and thus only of historic interest. During the past two decades, these subjects have been thoroughly transformed by cosmological observations and associated theory. For a fully up-to-date, pedagogical treatment, we recommend chapter 28 of MCP, or at a more elementary level, Schneider (2015). The most useful advanced textbook may be Weinberg (2008).

C. Cosmological observations over the past two decades suggest that *Chapter 30, Anisotropic and Homogeneous Cosmologies* is likely not relevant to the early evolution of our universe. However, it is of great importance for a fundamental new topic to be discussed below (see 4.D): the physical structure of singularities.

D. To this cosmological Part of *Gravitation*, we would add a major new topic (chapter): *Inflationary expansion in the very early universe*, as treated, for example, in Peacock (1999); Hobson, Efstathiou, and Lasenby (2006); Sasaki (2015); and section 28.7.1 of MCP.

4. **Part VII, Gravitational Collapse and Black Holes**, is surprisingly up to date, in large measure because it focuses on theory and says little about observations. However, a few new theoretical developments (some major) have emerged since 1973 that should be included in any year-long advanced course on general relativity.

   A. To *Chapter 31, Schwarzschild Geometry*, and *Chapter 32, Gravitational Collapse*, we would add nothing.

   B. To *Chapter 33, Black Holes*, we would add the following topics.

      a. Exercises to explore geodesic orbits around a Kerr black hole numerically, by integrating Hamilton's equations for the super-Hamiltonian (33.27c) (Levin and Perez-Giz 2008).

b. A discussion of quasinormal modes of a Kerr black hole, motivated by Exercise 33.14; see, for example, chapter 12 of Maggiore (2017). (The first hint of these modes was found by Vishveshwara, 1970, in the form of ringdown waves like those that LIGO has detected 45 years later. By 1973, when *Gravitation* was published, the concept of quasinormal modes was fully in hand along with the equations for computing them, but the first numerical computation of their complex eigenfrequencies and eigenfunctions, by Chandrasekhar and Detweiler, 1975, was still two years in the future.)

c. Spherical accretion onto a Schwarzschild black hole and accretion disks around a Kerr black hole: at least a few exercises as, for example, in MCP. These topics are touched on in Box 33.3 of *Gravitation*, but given their great astrophysical importance today, they deserve greater and more up-to-date detail; see, for example, the brief discussion in Straumann (2013), the longer discussion in Abramovicz and Fragile (2013), or the very detailed discussion in Meier (2012).

d. The Blandford-Znajek (1977) mechanism by which magnetic fields extract spin energy from black holes to power jets; see, for example, MCP. For far greater detail, see Thorne, Price, and MacDonald (1986), which emphasizes the relativity, and McKinney, Tchekhovskoy, and Blandford (2012), which emphasizes the astrophysics.

e. Some discussion of astronomical observations of black holes and their astrophysical roles in the universe; see, for example, Narayan and McClintock (2015) or for greater detail, Meier (2012) and Schneider (2015). (What remarkable developments there have been here, since *Gravitation* was published!)

f. Hawking radiation, the associated thermal atmosphere of a black hole, and black-hole thermodynamics (all of which were developed within a year of publication of *Gravitation*, in the wake of Stephen Hawking's and others' bringing quantum field theory in curved spacetime into a sufficiently mature form; see 1.B.c above). See, for example, Carroll (2004) for a moderately brief, pedagogical treatment, and Wald (1994) for greater detail.

C. To *Chapter 34, Global Techniques, Horizons, and Singularity Theorems*, we would add a new set of topics that have been explored using global techniques since *Gravitation* was published: wormholes and topological censorship; and closed timelike curves, chronology horizons, and chronology protection. See, for example, Everitt and Roman (2012) for a not very technical discussion with references to the most important literature or Friedman and Higuchi (2008) for greater technical detail.

D. To *Part VII* we would also add the equivalent of one more chapter on *The Physical Structure of Generic Singularities and the Interiors of Black Holes*. This chapter would include the following.

a. The material in Chapter 30 (3.C above) on the Kasner and Mixmaster solutions of Einstein's equation, plus a more detailed discussion of the Belinsky, Khalatnikov, and Lifshitz (BKL) analysis, which suggests there is a generic, spatially inhomogeneous variant of Mixmaster (p. 806 of *Gravitation*); also, a description of numerical relativity simulations (Garfinkle 2004; Lim et al. 2009) that prove this to be true and reveal some surprising twists missed by BKL.

b. Analyses that show that the inner horizons, $r = r_-$, of a Kerr or Reissner-Nordstrøm black hole (Fig. 34.4) are highly unstable and that material or radiation falling into the hole triggers these instabilities, converting the inner horizons into generic null singularities (Poisson and Israel 1990; Marolf and Ori 2012).

5. In Part VIII, Gravitational Waves, the chapters on the theory of the waves and their generation are largely up to date, but the chapter on their detection is extremely out of date. More specifically:

A. *Chapter 35, Propagation of Gravitational Waves*, is essentially up to date.

B. The topics covered in *Chapter 36, Generation of Gravitational Waves*, are essentially up to date, but they need to be augmented by the following.

a. An overview of gravitational wave sources that are likely to be observed in the next decade or two; see, for example, Buonanno and Sathyaprakash (2015); Creighton and Anderson (2011); or, for far greater detail, Maggiore (2017).

b. A sketch of the post-Newtonian expansion of the waves from compact binary stars (higher-order corrections to this chapter's quadrupolar analysis); see, for example, Straumann (2013); and for greater detail, Poisson and Will (2014) or Blanchet (2014). Exercise 39.15 of *Gravitation* could be a starting point for this.

c. A description of numerical relativity simulations of the inspiral and merger of black-hole binaries, and black-hole/neutron-star binaries, and their gravitational waves (e.g., Choptuik, Lehner, and Pretorius 2015; Maggiore 2017); also, the nonlinear dynamics of curved spacetime triggered by black-hole mergers (e.g., Owen et al. 2011; Scheel and Thorne 2013).

d. A sketch of the analysis that shows that early-universe inflation parametrically amplifies gravitational vacuum fluctuations coming off the big bang, to produce a spectrum of primordial gravitational waves; see, for example, Mukhanov (2005) or Maggiore (2017).

C. *Chapter 37, Detection of Gravitational Waves*, is highly out of date. Although nothing is wrong with this chapter, and it can be of conceptual value (particularly Sections 37.1–37.3), it focuses on vibrating mechanical detectors, which have largely been abandoned. So in a modern course, we would replace Sections 37.4–37.10 by the following.

a. An overview of the four types of detectors that are expected to open up four different gravitational-wave frequency bands in the next two decades: ground-based interferometers, such as LIGO, which have already opened the high-frequency band (10–10,000 Hz); space-based detectors, such as LISA, in which drag-free spacecraft track each other with laser beams, that are expected to open up the low-frequency band (periods of minutes to hours) in the next 15 or 20 years; pulsar timing arrays (PTAs), which are expected to open the very low frequency band (periods of a year to a few tens of years) in the coming decade; and so-called B-mode polarization patterns in the CMB, which are induced by primordial gravitational waves with periods of millions to billions of years (the extremely low frequency band) and which may be definitively measured in the next decade or so. See, for example, Berger et al. (2015) and Maggiore (2017).

b. Detailed analyses of (idealized) ground-based interferometers, space-based detectors, and PTAs (e.g., Creighton and Anderson 2011; Saulson 2017; MCP); and analyses of the influence of gravitational waves on CMB polarization (e.g., Maggiore 2017).

c. A summary of observations of gravitational waves, which in 2017 are solely those by LIGO, such as Abbott et al. (2016a).

D. This could be a good spot, in an advanced course, to present an overview of what is known about the nonlinear dynamics of vacuum, curved spacetime (e.g., Scheel and Thorne 2014)—most of which has already been mentioned above.

a. The chaotic spacetime dynamics near a generic Mixmaster (BKL) singularity (4.D.a).

b. The more gentle dynamics near a generic null singularity (4.D.b).

c. The phase transitions, critical behavior, and scaling that show up in (nongeneric) "critical" gravitational collapse; for example, Choptuik, Lehner, and Pretorius (2015).

d. The interacting "tidal tendices" and "frame-drag vortices" that generate the gravitational waves in black-hole collisions; for example, MCP, or for greater detail Scheel and Thorne (2014), or for still greater detail Owen et al. (2011).

e. Nonlinear, two-dimensional turbulence (energy cascades from small scales to large scales) triggered by mode–mode coupling in perturbations of a fast spinning black hole; see Yang, Zimmerman, and Lehner (2015).

We suspect that these just "scratch the surface" on nonlinear spacetime dynamics, and that a rich range of other phenomena will be discovered in the coming years.

6. **Part IX, Experimental Tests of General Relativity**, is all correct, but since *Gravitation* was published, rapidly improving technology and vigorous efforts by creative experimenters have moved the most accurate experimental tests from errors of a few percent to errors as small as one part in 100,000; so, obviously, a huge amount of updating is necessary.
   A. A modern course might simply follow the discussion of experimental tests in recent pedagogical references, such as Will (2014, 2015).
   B. Or it might do the following.
      a. Preserve the discussion of foundational tests in *Chapter 38*, augmented by an overview of the current status of those and related experiments from Will (2014, 2015).
      b. Preserve the pedagogical discussion of the post-Newtonian approximation and the parametrized post-Newtonian formalism in *Chapter 39*, augmented by the corresponding analysis for the orbital motion of compact binaries (a straightforward extension of Exercise 39.15).
      c. Preserve the analysis of solar system experiments in *Chapter 40*, augmented by an overview of the current status of those experiments as in Will (2014, 2015).
      d. Add discussion and some analyses of experimental tests in binary pulsars (e.g., Straumann 2013; Will 2014, 2015); and also experimental tests based on gravitational wave observations of binary black holes, for which expectations are discussed in Yunes and Siemens (2013) and in Gair et al. (2013), and results are just beginning to emerge from LIGO (e.g., Abbott et al. 2016b).

7. **Part X, Frontiers**, is a beautiful overview of some important ideas that occupied John Wheeler's attention in the era when we wrote this book with him.
   A. *Chapter 41, Spinors*, is an introduction to this important topic in mathematical physics—an introduction that mixes the deep mathematics with the intuitive, visual, and physical viewpoint that was John's hallmark. This chapter stands on its own, with no need for change.

B. The *Regge Calculus*, laid out so beautifully in *Chapter 42*, has played a powerful conceptual role in general relativity for decades, but has never (yet) become an effective tool for numerical computations.

C. *Superspace*, as treated in *Chapter 43*, has long been a powerful underpinning for some approaches to formulating laws of quantum gravity.

D. *Chapter 44, Beyond the End of Time*, describes prescient ideas on which John focused in the 1960s–1980s. It is of great historical import, and it contains ideas that continue to have influence.

We commend these chapters to readers, followed by a perusal of modern applications on the physics archive, https://arxiv.org.

Gravitation and these updates clearly constitute far more material than can be covered in a full year course, just as *Gravitation* by itself did in 1973, when first published. Today, as then, a teacher or student or reader will want to select which portions to focus on, and at what depth. But the above summary does convey what we think important and worthy of study in 2017.

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Above all, we are indebted to our mentor and coauthor, John Archibald Wheeler, who enticed us into the arena of general relativity six decades ago with his optimism, enthusiasm, and eagerness for adventure.

Many colleagues, friends, students—and students of students of students—have rewarded us by embracing this heavy tome. They have our sincere thanks. We hope they continue to appreciate the beauty of the ideas described in our book: the intellectual universe that Einstein opened for humanity more than 100 years ago. And we hope and expect that they will help others, in coming decades, to see the magnificence and subtlety of Nature through this window.

We relied on textbooks, as well as on John, as we struggled to learn general relativity in the 1950s and early 1960s. We thank the authors of those texts: Peter Bergmann, Christian Møller, Richard Tolman, John Synge, and Lev Landau and Evgeny Lifshitz. Beyond these, John Wheeler encouraged Misner to look at the 1955 text by André Lichnerowicz, which does reflect the then-current differential geometry. Much of Part III in *Gravitation* reflects Misner's efforts to help Wheeler restate the differential geometry that Misner had learned from his Notre Dame mathematics mentor Arnold Ross, his Princeton mathematics advisor Donald Spencer, and fellow Princeton graduate students.

For the research that has transformed this field, financial support was needed. On behalf of our colleagues as well as ourselves, we thank the program directors who targeted that support with great wisdom over the early years, particularly Joshua Goldberg (for the U.S. Air Force), and Harry Zapolsky and then Richard Isaacson (for the National Science

Foundation), followed by many others when the necessary investments got large. The impact of gravitational wave observations will be huge over the coming decades. The chief architect on the rocky course from small-scale R&D to the massively big collaborations required for success was Barry Barish, while Joseph Weber's pioneering insights, courage, and determination from his beginnings with Wheeler in 1956 should never be overlooked.

For helping bring this textbook to fruition in 1973, we are indebted to many people; see the original Acknowledgments on page li.

Until 2015, *Gravitation* continued to sell many hundred copies a year. Then, through a series of acquisitions of publishers by publishers by publishers, it wound up in the hands of Macmillan, which took it out of print. Through persistence, finesse, and firmness, Joan Winstein succeeded in extracting all rights to *Gravitation* from Macmillan, and to her we are deeply grateful.

We were fortunate that Princeton University Press eagerly embraced the idea of producing a new hardback printing at a remarkably low purchase price. We thank the superb staff at the Press, who have worked so effectively with us to bring this printing to fruition, particularly Peter Dougherty, Ingrid Gnerlich, Arthur Werneck, Karen Carter, Lisa Black, and Jessica Massabrook.

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