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Panconnectivity and edge-fault-tolerant pancyclicity of augmented cubes ☆

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Abstract

As an enhancement on the hypercube Q_n, the augmented cube AQ_n, proposed by Choudum and Sunitha [S.A. Choudum, V. Sunitha, Augmented cubes, Networks, 40(2) (2002), 71–84], not only retains some of the favorable properties of Q_n but also possesses some embedding properties that Q_n does not. For example, AQ_n contains cycles of all lengths from 3 to 2^n, but Q_n contains only even cycles. In this paper, we obtain two stronger results by proving that AQ_n contains paths, between any two distinct vertices, of all lengths from their distance to $2^n - 1$; and AQ_n still contains cycles of all lengths from 3 to 2^n when any $(2n - 3)$ edges are removed from AQ_n. The latter is optimal since AQ_n is $(2n - 1)$-regular.

Keywords: Interconnection network; Panconnected; Pancyclic; Fault-tolerant; Augmented cube

1. Introduction

It is well known that interconnection networks play an important role in parallel computing/communication systems. One of the central issues in evaluating a network is to study the graph embedding problem [3,4]. The graph embedding problem asks if a guest graph is a subgraph of a host graph, and an important benefit of graph embeddings is that we can apply existing algorithms for guest graphs to host graphs. This problem has attracted a burst of studies in recent years. Cycle networks are suitable for designing simple algorithms with low communication costs. Since some parallel applications, such as those in image and signal processing, are originally designated on a cycle architecture, it is important to have effective cycle embedding in a network. The cycle embedding properties of many interconnection networks have been investigated in the literature (see, for example, [1,6,11,14,18,20–22]).

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Edge and/or vertex failures are inevitable when a large parallel computer system is put in use. Therefore, the fault-tolerant capacity of an interconnection network is a critical issue in parallel computing. Fault-tolerant properties have been widely studied in many networks, such as [2,7–10,12,15,17,19].

It is well known that the hypercube has been one of the most popular interconnection networks for parallel computer/communication system. This is partly due to its attractive properties such as regularity, recursive structure, node and edge symmetry, maximum connectivity, as well as effective routing and broadcasting algorithms [13].

As an enhancement on the hypercube Q_n, the augmented cube AQ_n, proposed by Choudum and Sunitha [5], not only retains some of the favorable properties of Q_n but also possesses some embedding properties that Q_n does not. For example, AQ_n contains cycles of all lengths from 3 to 2^n, but Q_n contains only even cycles.

In this paper, we obtain two stronger results: AQ_n contains paths, between any two distinct vertices, of all lengths from their distance to $2^n - 1$; and AQ_n still contains cycles of all lengths from 3 to 2^n when any $(2n - 3)$ edges are removed from AQ_n. The latter is optimal since AQ_n is $(2n - 1)$-regular.

The rest of this paper is organized as follows. Section 2 gives some basic definitions used in our discussion. The proofs of our main results are in Section 3 and in Section 4. Some conclusions are given in Section 5.

2. Basic definitions

An interconnection network is usually represented by an undirected simple graph $G = (V, E)$, where V and E are the vertex set and the edge set, respectively, of G. In this paper, we use a graph and a network interchangeably. For graph terminology and notation not defined here we follow [16].

Two vertices u and v are adjacent if $(u, v) \in E(G)$. A path is a finite sequence of adjacent vertices, written as $\langle u, u_1, \ldots, v\rangle$, in which all the vertices $u, u_1, \ldots, v$ are distinct except possibly $u = v$. A path joining u and v is called a uv-path, and the distance between u and v is the length of a shortest uv-path, denoted by $d_G(u, v)$, or simply $d(u, v)$. The diameter $D(G)$ of G is the maximum distance between any two vertices of G. A path is called a hamiltonian path if it contains every vertex of G exactly once. A uv-path of length ℓ is denoted by $P_\ell(u, v) = \langle u, u_1, \ldots, v\rangle$, where the vertices u and v are end vertices of P and ℓ is the number of edges in P. $P_\ell(u, v)$ is called a cycle of length ℓ if $u = v$ and ℓ is at least three. We use C_ℓ to denote a cycle of length ℓ. A cycle is called a hamiltonian cycle of G if it contains every vertex of G exactly once. A graph G is hamiltonian if G contains a hamiltonian cycle.

A graph G is pancyclic if it contains a cycle of length ℓ for each ℓ with $3 \leqslant \ell \leqslant |V|$. A graph G is hamiltonian connected if there exists a hamiltonian path joining any two vertices of G. A graph G is panconnected if for any two distinct vertices u and v of G and for each integer ℓ with $d(u, v) \leqslant \ell \leqslant |V| - 1$, there is a uv-path of length ℓ in G. If a graph G is panconnected then clearly it is hamiltonian connected and pancyclic.

A graph G is k (respectively, k-edge)-fault-tolerant hamiltonian if $G - F$ is still hamiltonian for any $F \subseteq E(G) \cup V(G)$ (respectively, $F \subseteq E(G)$) with $|F| \leqslant k$ [8]. Similarly, k-fault-tolerant hamiltonian connected graphs and k-edge-fault-tolerant pancyclic graphs can be defined.

The n-dimensional augmented cube AQ_n $(n \geqslant 1)$ can be defined recursively as follows: AQ_1 is a complete graph K_2 with the vertex set $\{0, 1\}$. For $n \geqslant 2$, AQ_n is obtained by taking two copies of the augmented cube AQ_{n-1}, denoted by AQ^0_{n-1} and AQ^1_{n-1}, and adding $2 \times 2^{n-1}$ edges between the two as follows.

Let $V(AQ^0_{n-1}) = \{0u_{n-1} \ldots u_2u_1 : u_i = 0 \quad \text{or } 1\}$ and $V(AQ^1_{n-1}) = \{1u_{n-1} \ldots u_2u_1 : u_i = 0 \quad \text{or } 1\}$. A vertex $u = 0u_{n-1} \ldots u_2u_1$ of AQ^0_{n-1} is joined to a vertex $v = 1v_{n-1} \ldots v_2v_1$ of AQ^1_{n-1} if and only if either

(i) $u_i = v_i$ for $1 \leqslant i \leqslant n - 1$; in this case, v (respectively, u) is called a hypercube neighbor of u (respectively, v), setting $v = u^h$ or $u = v^h$, or
(ii) $u_i = \bar{v}_i$ for $1 \leqslant i \leqslant n - 1$; in this case, v (respectively, u) is called a complement neighbor of u (respectively, v), setting $v = u^c$ or $u = v^c$.

The graphs shown in Fig. 1 are the augmented cubes AQ_1, AQ_2 and AQ_3, respectively.

Obviously, AQ_n is a $(2n - 1)$-regular graph with 2^n vertices.

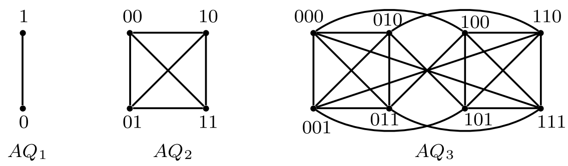


Fig. 1. Three augmented cubes AQ_1, AQ_2 and AQ_3

3. Panconnectivity of augmented cubes

According to the definition of augmented cubes, we write this recursive construction of AQ_n symbolically as $AQ_n = L \oplus R$, where $L \cong AQ^0_{n-1}$ and $R \cong AQ^1_{n-1}$. We call the edges between L and R crossed edges, denoted by E_c. Clearly every vertex of AQ_n is incident with two crossed edges. The following properties are derived directly from the definition.

Property 1. *If* $(u, v) \in E(L)$, *then* $(u^h, v^h) \in E(R)$ *and* $(u^c, v^c) \in E(R)$. *For any two distinct vertices* $u \neq v$ *in* L, $u^h \neq v^h$ *and* $u^c \neq v^c$.

Property 2. *For any vertex* $u \in V(L)$, $(u^h, u^c) \in E(R)$. *If* $v = u^h \in V(R)$, *then the subgraph in* AQ_n *induced by* $\{u, u^c, v, v^c\}$ *is a complete graph* K_4.

Although many interconnection networks have been shown to be hamiltonian or pancyclic, only a few of them have been shown to be panconnected. In this section, we show that the augmented cube is panconnected. The following result, which can be found in [5], is useful for us.

Lemma 1. *Let* u *and* v *be any two vertices in* AQ_n *with* $n \geqslant 2$. *Then* $d_{AQ_n}(u, v) = d_L(u, v)$ *if both* u *and* v *are in* L. *Similarly,* $d_{AQ_n}(u, v) = d_R(u, v)$ *if both* u *and* v *are in* R. *If* $u \in L$ *and* $v \in R$, *then there exist a shortest* uv*-path* P_1 *in* AQ_n *with all its vertices (except* v*) in* L *and a shortest* uv*-path* P_2 *in* AQ_n *with all its vertices (except* u*) in* R.

Theorem 1. *For any integer* $n \geqslant 1$, *the augmented cube* AQ_n *is panconnected.*

Proof. We prove the theorem by induction on $n \geqslant 1$. Obviously, AQ_1 and AQ_2 are panconnected (see Fig. 1). Assume that AQ_{n-1} is panconnected for $n \geqslant 3$. We now consider the graph AQ_n. Let u and v be any two vertices in $AQ_n = L \oplus R$. We will prove that there is a uv-path of length ℓ, for each ℓ with $d_{AQ_n}(u, v) \leqslant \ell \leqslant 2^n - 1$. Consider the following two cases.

Case 1 Both u and v are in L or R. Without loss of generality, we may assume both u and v are in L.

For $d_{AQ_n}(u, v) \leqslant \ell \leqslant 2^{n-1} - 1$, according to Lemma 1, $d_{AQ_n}(u, v) = d_L(u, v)$. By the induction hypothesis, there exists a uv-path of length ℓ in L, also in AQ_n.

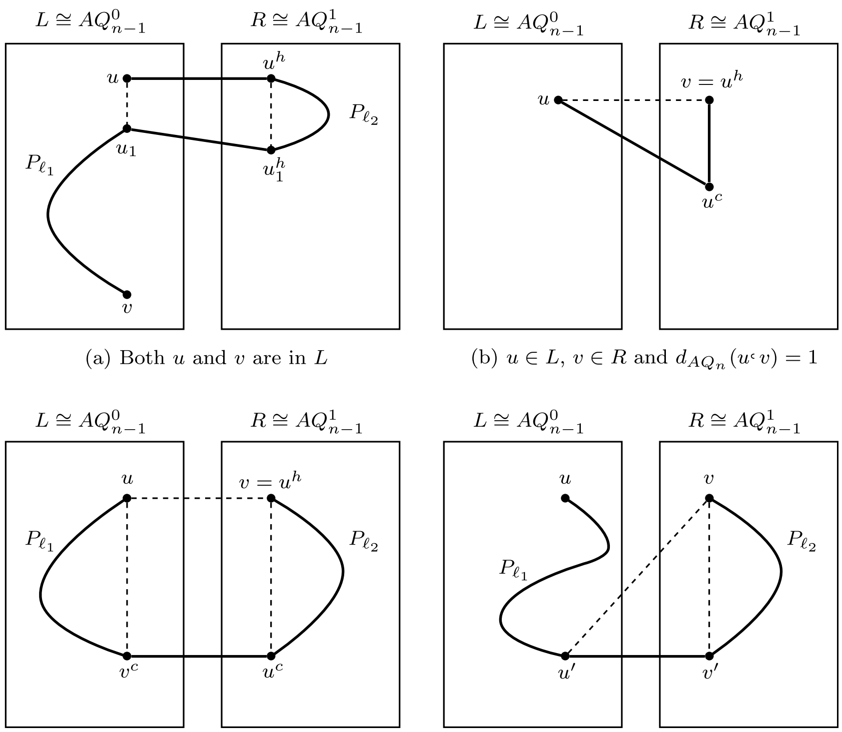
For $2^{n-1} \leqslant \ell \leqslant 2^n - 1$, we can write $\ell = \ell_1 + \ell_2 + 1$ where $D(L) = \lceil \frac{n-1}{2} \rceil < 2^{n-1} - 2 \leqslant \ell_1 \leqslant 2^{n-1} - 1$ and $1 \leqslant \ell_2 \leqslant 2^{n-1} - 1$. Since $d_L(u, v) \leqslant D(L)$, by the induction hypothesis, there exists a uv-path of length ℓ_1 in L. Let $P_{\ell_1} = \langle u, u_1, \ldots, v\rangle$ be a uv-path of length ℓ_1 in L. Let u^h and u_1^h be the hypercube neighbors of u and u_1 in R, respectively. By Property 1, $(u^h, u_1^h) \in E(R)$, i.e., $d_R(u^h, u_1^h) = 1$. By the induction hypothesis, there is a $u^h u_1^h$-path P_{ℓ_2} of length ℓ_2 in R. Hence $P = \langle u, u^h, P_{\ell_2}, u_1^h, u_1, \ldots, v\rangle$ is a uv-path of length ℓ in AQ_n (see Fig. 2a).

Case 2 $u \in L$ and $v \in R$.

Subcase 2.1 $d_{AQ_n}(u, v) = 1$. Then $v = u^c$ or $v = u^h$. Without loss of generality, we assume $v = u^h$. By Property 2, we have $(u^h, u^c) \in E(R)$. Then $P = \langle u, u^c, u^h = v\rangle$ is a uv-path of length 2 in AQ_n (see Fig. 2b).

For $3 \leqslant \ell \leqslant 2^n - 1$, we can write $\ell = \ell_1 + \ell_2 + 1$ where $1 \leqslant \ell_1 \leqslant 2^{n-1} - 1$ and $1 \leqslant \ell_2 \leqslant 2^{n-1} - 1$. Note that $v^h = u$; by Property 2, $(u, v^c) \in E(L)$ and $(v^c, u^c) \in E(AQ_n)$. By the induction hypothesis, there exist a uv^c-path P_{ℓ_1} of length ℓ_1 in L and a $u^c v$-path P_{ℓ_2} of length ℓ_2 in R. Then $P = \langle u, P_{\ell_1}, v^c, u^c, P_{\ell_2}, v\rangle$ is a uv-path of length ℓ in AQ_n (see Fig. 2c).

Subcase 2.1 $d_{AQ_n}(u, v) \geqslant 2$. By Lemma 1, there exists a shortest uv-path $P = \langle u, \ldots, u', v\rangle$ in AQ_n with all its vertices (except v) in L. Let $P_L = \langle u, \ldots, u'\rangle$ be the segment of path P in L. Then the length of P_L is $d(u, v) - 1$ and v is a neighbor of u' in R. Assume the other neighbor of u' in R is v'; by Property 2, we have $(v', v) \in E(R)$.



(a) Both u and v are in L

(b) $u \in L$, $v \in R$ and $d_{AQ_n}(u^c v) = 1$

(c) $u \in L$, $v \in R$ and $d_{AQ_n}(u^c v) = 1$

(d) $u \in L$, $v \in R$ and $d_{AQ_n}(u^c v) \geq 2$

Fig. 2. Illustrations for the proof of Theorem 1. (A straight line or a dashed line represents an edge and a curve line represents a path between two vertices.)

For $d_{AQ_n}(u,v) + 1 \leqslant \ell \leqslant 2^n$, we can write $\ell = \ell_1 + \ell_2 + 1$ where $d_{AQ_n}(u,v) - 1 \leqslant \ell_1 \leqslant 2^{n-1} - 1$ and $1 \leqslant \ell_2 \leqslant 2^{n-1} - 1$. By the induction hypothesis, there exist a uu'-path P_{ℓ_1} of length ℓ_1 in L and a $v'v$-path P_{ℓ_2} of length ℓ_2 in R. Then $P = \langle u, P_{\ell_1}, u', v', P_{\ell_2}, v\rangle$ is a uv-path of length ℓ in AQ_n (see Fig. 2d). □

4. Edge-fault-tolerant pancyclicity

Let $F \subset E(AQ_n)$. An edge (u,v) is called a faulty edge if $(u,v) \in F$. A subgraph H of AQ_n is called fault-free if H contains no faulty edges (i.e., $E(H) \cap F = \emptyset$). For convenience of discussion, we define the following subsets of F: $F_L = F \cap E(L)$, $F_R = F \cap E(R)$ and $F_c = F \cap E_c$. Note that $F = F_L \cup F_R \cup F_c$.

The following results proved in [10] are useful in the proof of Theorem 2.

Lemma 2. *Let $\{u,v,x,y\}$ be any four distinct vertices of $AQ_n (n \geqslant 2)$. Then there exist a ux-path and a vy-path such that they are disjoint and contain all vertices of AQ_n.*

Lemma 3. *The augmented cube AQ_n is $(2n-3)$-fault-tolerant hamiltonian and $(2n-4)$-fault-tolerant hamiltonian connected for any integer $n \notin \{1,3\}$.*

The above lemma states that with up to $(2n-3)$ faulty edges and faulty vertices, $AQ_n (n \notin \{1,3\})$ still contains a hamiltonian cycle, and with up to $(2n-4)$ faulty edges and faulty vertices, $AQ_n (n \notin \{1,3\})$ is still hamiltonian connected. It is shown in [10] that there are 3 faulty vertices F in AQ_3 such that $AQ_3 - F$ is non-hamiltonian and there are 2 faulty vertices F in AQ_3 such that $AQ_3 - F$ is non-hamiltonian connected. If the faulty elements contain no vertices, we prove the following two lemmas. The proofs of these lemmas are omitted here since they can be directly verified in a straight forward manner as AQ_3 contains just 8 vertices.

Lemma 4. *The augmented cube AQ_3 is 2-edge-fault-tolerant hamiltonian connected.*

Lemma 5. *The augmented cube AQ_3 is 3-edge-fault-tolerant pancyclic, hence it is 3-edge-fault-tolerant hamiltonian.*

Theorem 2. *The augmented cube AQ_n is $(2n-3)$-edge-fault-tolerant pancyclic for any integer $n \geqslant 2$.*

Proof. We prove the theorem by induction on $n \geqslant 2$. Obviously, AQ_2 is 1-edge-fault-tolerant pancyclic since AQ_2 is a complete graph K_4. By Lemma 5, the conclusion is true for AQ_3. Assume that the theorem is true for AQ_{n-1} with $n \geqslant 4$. We now consider AQ_n. Let $F \subset AQ_n$ be a set of faulty edges in $AQ_n = L \oplus R$ with $|F| = 2n-3$. Without loss of generality, we may assume $|F_L| \geqslant |F_R|$. We will prove that there is a cycle of length ℓ for each ℓ with $3 \leqslant \ell \leqslant 2^n$ in $AQ_n - F$. Consider the following three cases.

Case 1 $|F_L| \leqslant 2n-5$. Then $|F_R| \leqslant 2n-6$ for $n \geqslant 4$, because $|F_L| \geqslant |F_R|$ and $|F_L| + |F_R| \leqslant 2n-3$.

For $3 \leqslant \ell \leqslant 2^{n-1}$, by the induction hypothesis R is $(2n-5)$-edge-fault-tolerant pancyclic, and so there is a cycle of length ℓ in $R - F_R$ since $|F_R| \leqslant 2n-6$.

For $\ell = 2^{n-1}+1$, since $2^{n-1} > 2n-3$, there is a vertex u in L such that the two crossed edges (u, u^h) and (u, u^c) are both fault-free. By Lemmas 3 and 4, $R - F_R$ is still hamiltonian connected. There is a hamiltonian $u^h u^c$-path P_R^h in $R - F_R$. Then $C = \langle u, u^h, P_R^h, u^c, u \rangle$ is a fault-free cycle of length $2^{n-1}+1$ (see Fig. 3a).

For $2^{n-1}+2 \leqslant \ell \leqslant 2^n$, we can write $\ell = 2^{n-1}+1+\ell_1$, where $1 \leqslant \ell_1 \leqslant 2^{n-1}-1$. By Lemmas 3 and 4, R is $(2n-6)$-edge-fault-tolerant hamiltonian connected and L is $(2n-5)$-edge-fault-tolerant hamiltonian. Thus there is a hamiltonian cycle $C = \langle u_0, u_1, \ldots, u_{2^{n-1}-1}, u_0 \rangle$ in $L - F_L$. We claim that there exists a $u_i u_j$-path P_{ℓ_1} of length ℓ_1 on the cycle C such that $(j-i)(\bmod 2^{n-1}) = \ell_1$ (that is $j - i = \ell_1 + k \cdot 2^{n-1}$ where $k \in \{0, 1, -1\}$) and one of the two sets of edges $\{(u_i, u_i^h), (u_j, u_j^h)\}$, $\{(u_i, u_i^c), (u_j, u_j^c)\}$ is fault-free. For every vertex u_i on the cycle C, there are two different paths $u_i u_j$-path and $u_i u_{j'}$-path both of length ℓ_1. Hence, there are 2^{n-1} different paths of length ℓ_1 on the cycle C. Suppose to the contrary that there do not exist such u_i and u_j. Then there are at least 2^{n-2} faults in $\{(u_i, u_i^h), i = 0, 1, \ldots, 2^{n-1}-1\}$ and at least 2^{n-2} faults in $\{(u_i, u_i^c), i = 0, 1, \ldots, 2^{n-1}-1\}$. Thus there are at least 2^{n-1} faults outside L. However $2^{n-1} > 2n-3$ for $n \geqslant 4$, and so we obtain a contradiction. Hence, there exist such two vertices u_i and u_j. Without loss of generality, assume $\{(u_i, u_i^h), (u_j, u_j^h)\}$ are fault-free. Since $R - F_R$ is hamiltonian connected, there is a hamiltonian $u_i^h u_j^h$-path P_R^h in $R - F_R$. Then $C = \langle u_i, P_{\ell_1}, u_j, u_j^h, P_R^h, u_i^h, u_i \rangle$ is a cycle of length ℓin $AQ_n - F$ (see Fig. 3b).

Case 2 $|F_L| = 2n-4$. Then $|F_R| \leqslant 1$ and $|F_c| \leqslant 1$.

For $3 \leqslant \ell \leqslant 2^{n-1}+1$, we can construct a cycle of length ℓ similar to as in *Case* 1.

For $2^{n-1}+2 \leqslant \ell \leqslant 2^n$, we can write $\ell = 2^{n-1}+1+\ell_1$, where $1 \leqslant \ell_1 \leqslant 2^{n-1}-1$. By Lemmas 3 and 4, R is $(2n-6)$-edge-fault-tolerant hamiltonian connected and L is $(2n-5)$-edge-fault-tolerant hamiltonian. Thus there is a hamiltonian path $P_L^h = \langle u_0, u_1, \ldots, u_{2^{n-1}-1} \rangle$ in $L - F_L$. For any two vertex u_0 and u_i on P_L^h, one of the two sets of edges $\{(u_0, u_0^h), (u_i, u_i^h)\}$, $\{(u_0, u_0^c), (u_i, u_i^c)\}$ is fault-free since $|F_c| \leqslant 1$. Fix $i = \ell_1$. Without loss of generality, assume $\{(u_0, u_0^h), (u_i, u_i^h)\}$ is fault-free. Since $R - F_R$ is hamiltonian connected, there is a hamiltonian $u_0^h u_i^h$-path P_R^h in $R - F_R$. Then $C = \langle u_0, P_i, u_i, u_i^h, P_R^h, u_0^h, u_0 \rangle$ is a cycle of length ℓ in $AQ_n - F$ (see Fig. 3c).

Case 3 $|F_L| = 2n-3$. The faulty edges are all in L.

For $3 \leqslant \ell \leqslant 2^{n-1}+1$, we can construct a cycle of length ℓ similar as in *Case* 1.

For $2^{n-1}+2 \leqslant \ell \leqslant 2^n$, if there is a fault-free hamiltonian path in L, we can construct the required cycles with the method similar to that of *Case* 2. Thus suppose there does not exist a fault-free hamiltonian path in L. We can mark any two edges (u_1, v_1) and (u_2, v_2) in F_L as temporarily fault-free. By the induction hypothesis applied to this amended L, there is a hamiltonian cycle C in $L - \{F_L - \{(u_1, v_1), (u_2, v_2)\}\}$ and both (u_1, v_1) and (u_2, v_2) are on the cycle C (if at least one of (u_1, v_1) and (u_2, v_2) is not on the cycle, then there is a fault-free hamiltonian path in the original L). Thus $C - \{(u_1, v_1), (u_2, v_2)\}$ contains two fault-free paths P_1 and P_2 in the original L such that $P_1 \cup P_2$ span L. We denote the length of P_1 and P_2 as ℓ' and ℓ'', respectively. We may assume $\ell' \leqslant \ell''$.

Subcase 3.1 All faulty edges are incident with a vertex u. In this case the path P_1 is only a vertex u, hence $\ell' = 0$ and $\ell'' = 2^{n-1}-2$.

For $2^{n-1}+2 \leqslant \ell \leqslant 2^n - 1$, we can construct the required cycle using the path P_2 similarly as in *Case* 2.

For $\ell = 2^n$, since $2n-3 > 3$ for $n \geqslant 4$, we can choose two faulty edges (u, v_1) and (u, v_2) such that the neighbors u^h and u^c of u in R are not adjacent with v_1 and v_2. By Lemma 2, there exist a $u^h v_1^h$-path P_{R_1} and a

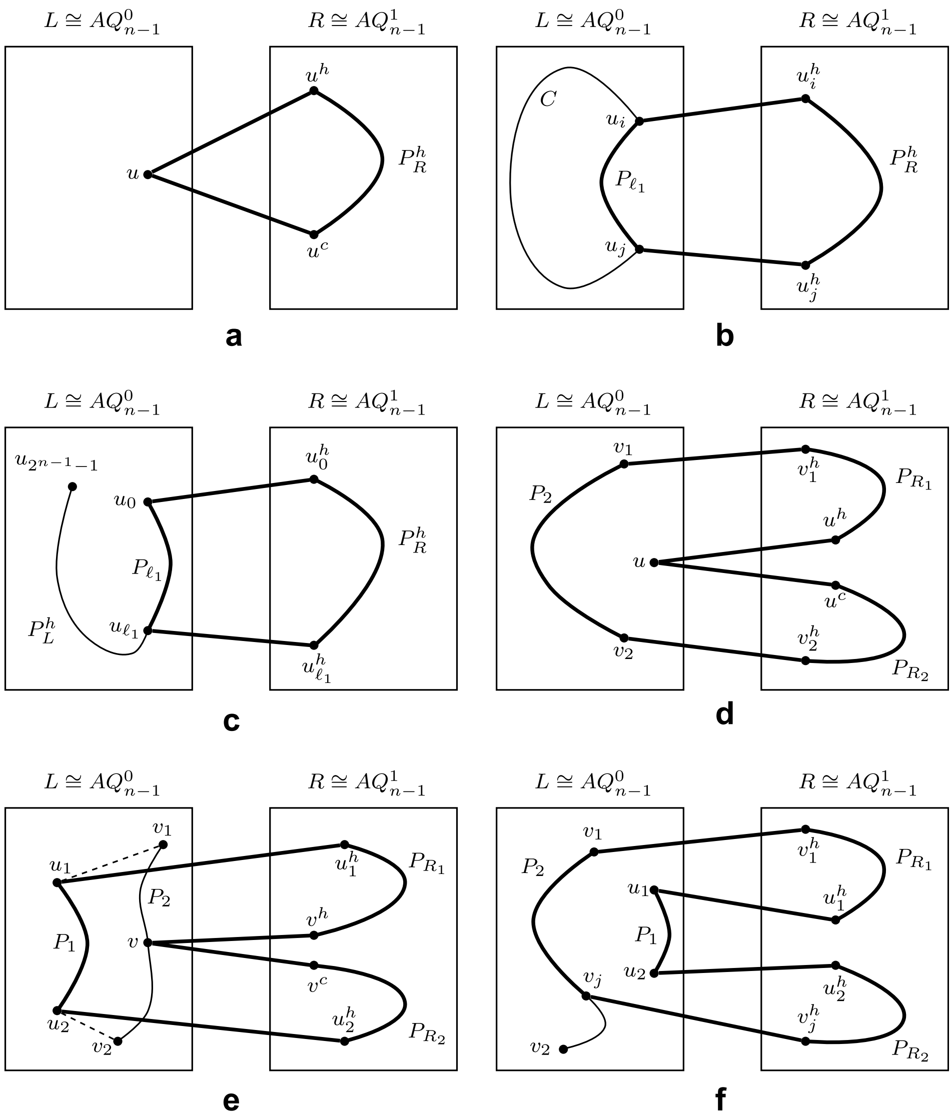


Fig. 3. Illustrations for the proof of Theorem 2. (A straight line represents an edge and a curve line represents a path between two vertices.)

$u^c v_2^h$-path P_{R_2} such that they are disjoint and contain all vertices of R. Then $C = \langle u, u^h, P_{R_1}, v_1^h, v_1, P_2, v_2, v_2^h, P_{R_2}, u^c, u\rangle$ is a cycle of length 2^n in $AQ_n - F$ (see Fig. 3d).

Subcase 3.2 There are two edges (u_1, v_1) and (u_2, v_2) in F_L such that they are not adjacent (remember, $2n - 3 > 3$). Using these two edges, our two paths P_1 and P_2 in $L - F_L$ non-trivial and such that $P_1 \cup P_2$ span L. Then $\ell' \geqslant 1$ and $\ell'' \geqslant 2^{n-2} - 1 > 2$. Without loss of generality, we may assume P_1 is a u_1u_2-path and P_2 is a v_1v_2-path.

For $2^{n-1} + 2 \leqslant \ell \leqslant 2^{n-1} + \ell' + 1$, we can construct the required cycle using the path P_1 with the method similar to that of *Case* 2.

For $\ell = 2^{n-1} + \ell' + 2$, since the length of P_2 is greater than 2, there are at least 3 vertices on P_2. There exists a vertex v on P_2 such that the neighbors v^h and v^c of v in R are not adjacent with the vertices u_1 and u_2. By Lemma 2, there exist a $u_1^h v^h$-path P_{R_1} and a $u_2^h v^c$-path P_{R_2} such that they are disjoint and contain all the vertices of R. Then $C = \langle u_1, u_1^h, P_{R_1}, v^h, v, v^c, P_{R_2}, u_2^h, u_2, P_1, u_1\rangle$ is a cycle of length $2^{n-1} + \ell' + 2$ in $AQ_n - F$ (see Fig. 3e).

For $2^{n-1} + \ell' + 3 \leqslant \ell \leqslant 2^n$, we can write $\ell = 2^{n-1} + \ell' + 2 + \ell_1$ where $1 \leqslant \ell_1 \leqslant 2^{n-1} - 3$. Note that $\ell_1 = \ell - 2^{n-1} - \ell' - 2 \leqslant 2^{n-1} - \ell' - 2 = \ell''$. Choose v_1w-path P_{ℓ_1} of length ℓ_1 on the path P_2. By Lemma 2, there exist a $u_1^h v_1^h$-path P_{R_1} and a $u_2^h w^h$-path P_{R_2} such that they are disjoint and contain all vertices of R. Then $C = \langle u_1, u_1^h, P_{R_1}, v_1^h, v_1, P_{\ell_1}, v_j, v_j^h, P_{R_2}, u_2^h, u_2, P_1, u_1 \rangle$ is a cycle of length ℓ in $AQ_n - F$ (see Fig. 3f).

The theorem follows. □

5. Conclusions

Linear arrays (paths) and rings (cycles), two of the most fundamental networks for parallel and distributed computation, are suitable for developing simple algorithms with low communication cost. The fault-tolerant pancyclicity of an interconnection network is a measure of its capability of implementing ring-structured parallel algorithms in a communication-efficient fashion in the presence of faults. In this paper, we prove that every augmented cube AQ_n is panconnected. In other words, for any two distinct vertices u and v of AQ_n and for each integer ℓ with $d(u,v) \leqslant \ell \leqslant 2^n - 1$, there is a uv-path of length ℓ in AQ_n. We also show that the augmented cube AQ_n is $(2n-3)$-edge-fault-tolerant pancyclic. This result is optimal since AQ_n is $(2n-1)$-regular.

In view of the fact that hypercube networks are not pancyclic, augmented cubes are superior to hypercubes in terms of the panconnectivity and fault-tolerant pancyclicity. Our further work is to determine the pancyclicity of augmented cubes in the presence of hybrid faults.

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